

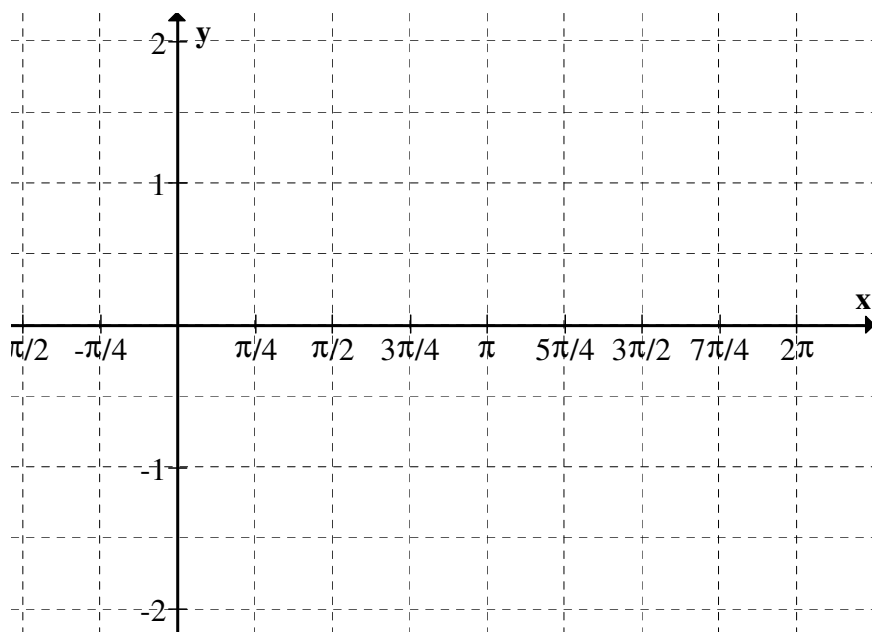
S5.1 & 5.3 GRAPHS & TRANSFORMATIONS OF SINE, COSINE, AND TANGENT FUNCTIONS

KEY CONCEPTS

From the previous unit, you can construct a table of values of the trigonometric functions for the multiples of $\pi/4$, or 45° , between 0 and 2π inclusive.

Angle, x	$y = \sin x$	$y = \cos x$	$y = \tan x$
0			
$\frac{\pi}{4}$			
$\frac{\pi}{2}$			
$\frac{3\pi}{4}$			
π			
$\frac{5\pi}{4}$			
$\frac{3\pi}{2}$			
$\frac{7\pi}{4}$			
2π			

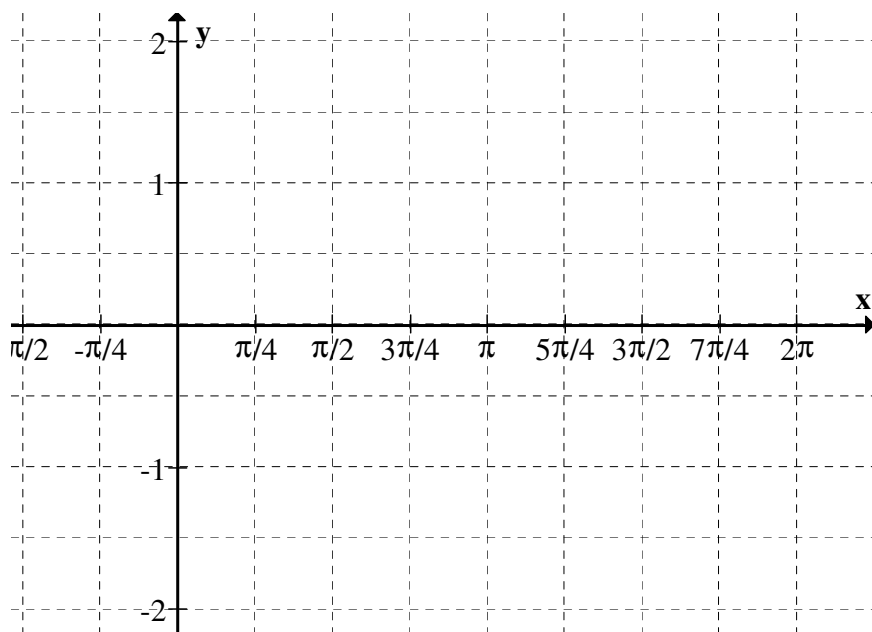
Sketch the graph of $y = \sin x$ between $x = 0$ and $x = 2\pi$.



Period = _____

Amplitude = _____

Sketch the graph of $y = \cos x$ between $x = 0$ and $x = 2\pi$.

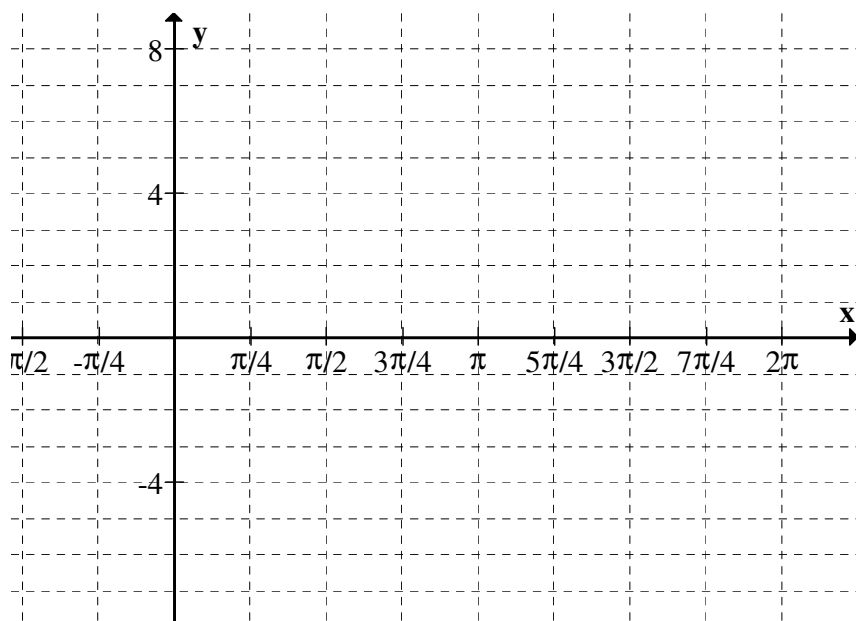


Period = _____

Amplitude = _____

Recall that $\tan x = \frac{\sin x}{\cos x}$. The tangent graph has a vertical asymptote anywhere cosine equals zero.

Sketch the graph of $y = \tan x$ between $x = 0$ and $x = 2\pi$. Sketch the vertical asymptotes as dotted lines.



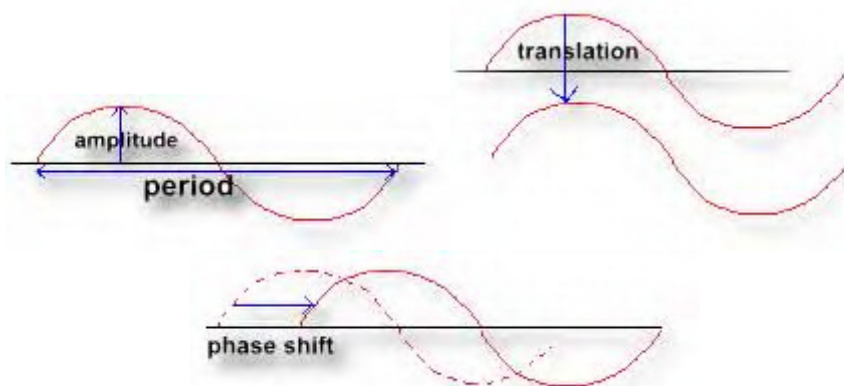
Period = _____

Amplitude = _____

Notice that the graph of $\tan x$ between $x = \pi$ and $x = 2\pi$ is a repeat of the graph of tangent between $x = 0$ and $x = \pi$. The period of tangent, unlike sine, cosine, cosecant and secant, is π rather than 2π . Thus $\tan(x + \pi) = \tan x$ for all x in the domain of the tangent function.

These graphs, like any other graph of a function, can be transformed. The table below outlines each change for each trig. ratio.

These definitions may help:



The transformation of a sine or cosine function $f(x)$ to $g(x)$ has the general form

$$g(x) = a f[k (x - d)] + c$$

where $|a|$ is the amplitude,
 d is the phase shift,
 & c is the vertical translation.

The period is given by $\frac{2\pi}{k}$.

If $a < 0$, there is a reflection in the x-axis.

Examples:

1. $Y = 3 \cos x$

What is the amplitude?

2. $Y = -4 \cos x$

What is the amplitude?

3. $Y = 3 \sin 2x$

What is the period in degrees?

4. $Y = \sin 10x$

What is the period in degrees?

5. $Y = 2 \tan 5x$

What is the amplitude?

6. $Y = 3 \cos (x - 30)$

What is the phase shift?

7. $Y = -5 \sin 3(x + 20)$

What is the period in degrees?

8. $Y = 6 \cos 2(x + 60) + 1$

What is the phase shift?

9. $Y = 6 \tan 2x$

What is the period in degrees?

10. $Y = 2 \cos (5x - 60)$

What is the phase shift?

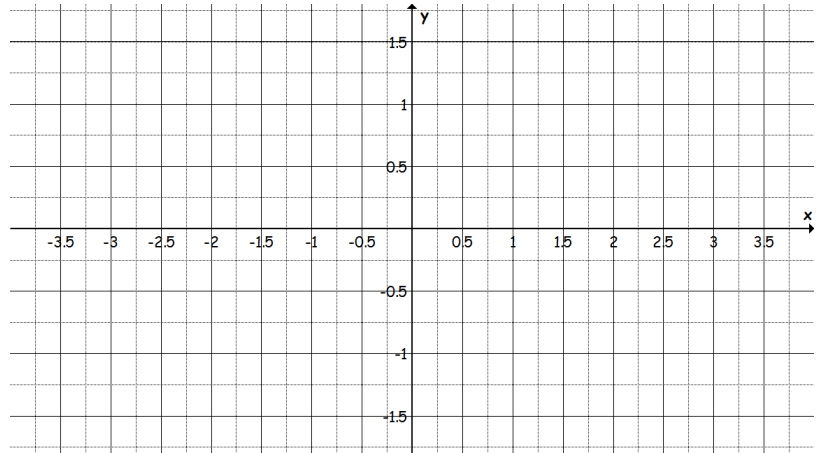
11. $Y = 2 \sin 2(x + 10) + 3$

What is the vertical translation?

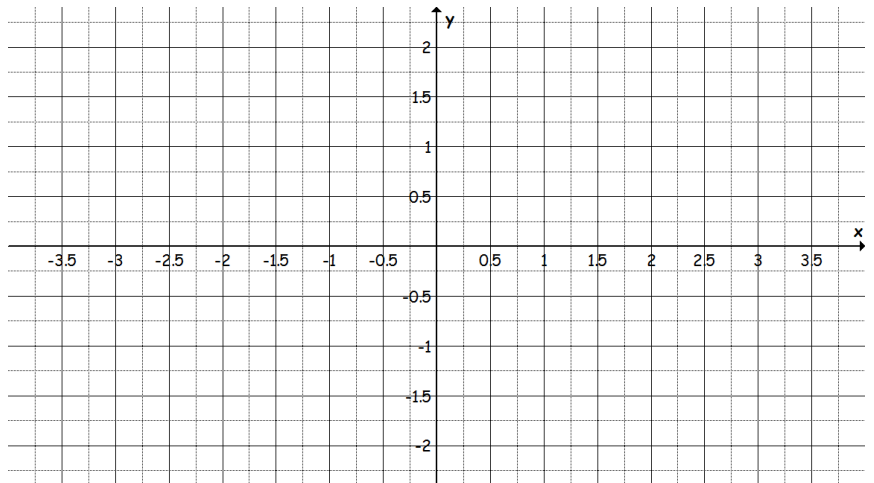
12. $Y = 3 \tan (2x - 60) - 3$

What is the period in degrees?

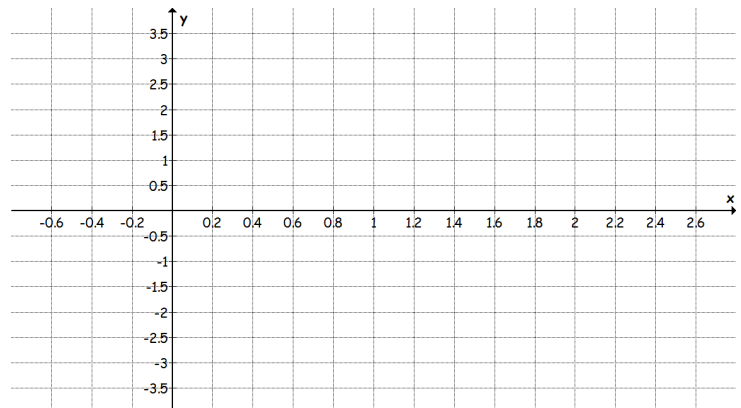
13. Sketch $y = 1.5\sin(2x + 6)$.



14. Sketch $y = -2\cos(2x + 3)$.



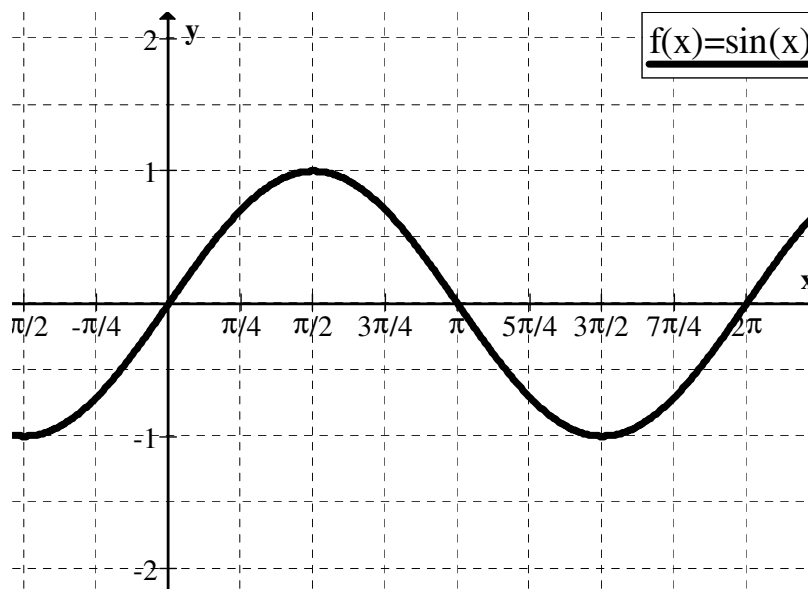
15. Transform the function $f(x) = \sin x$ to $g(x)$ such that $g(x)$ has an amplitude of 3, a period of $\frac{3\pi}{2}$, a phase shift of $\frac{\pi}{6}$ to the left, and a vertical translation of 2 units downward.



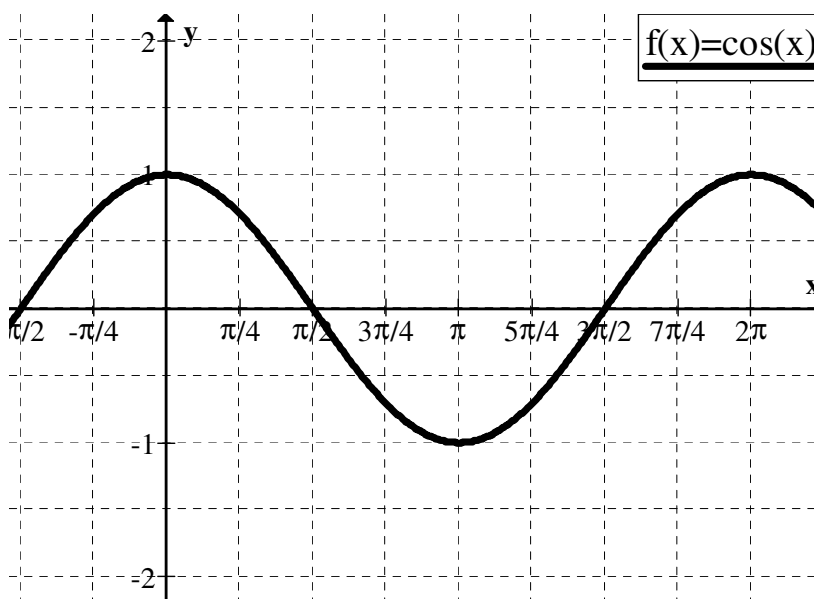
S5.2 GRAPHS OF RECIPROCAL TRIGONOMETRIC FUNCTIONS

KEY CONCEPTS

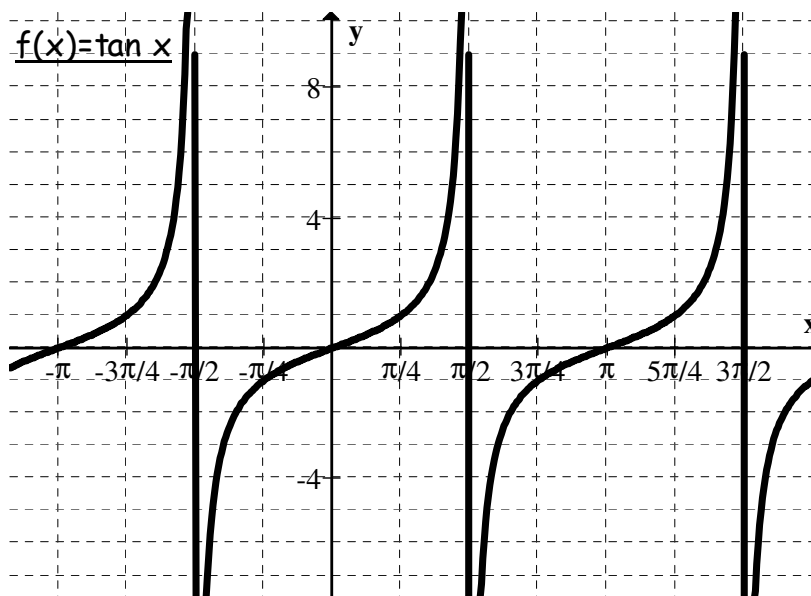
Below is the graph of $y = \sin x$. Recalling that $\csc x = \frac{1}{\sin x}$, sketch the graph of $y = \csc x$ between $x = 0$ and $x = 2\pi$.



Below is the graph of $y = \cos x$. Recalling that $\sec x = \frac{1}{\cos x}$, sketch the graph of $y = \sec x$ between $x = 0$ and $x = 2\pi$.



Below is the graph of $y = \tan x$. Recalling that $\cot x = \frac{1}{\tan x}$, sketch the graph of $y = \cot x$ between $x = 0$ and $x = 2\pi$.



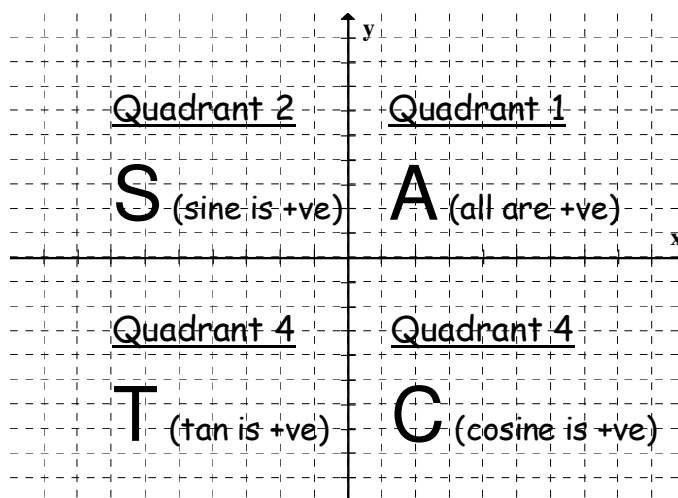
Copy and complete the summary table.

Property	$y = \csc x$	$y = \sec x$	$y = \cot x$
Domain			
Range			
Period			
Equations of asymptotes on the interval $x \in [0, 2\pi]$			

S5.4

SOLVE TRIGONOMETRIC EQUATIONS**KEY CONCEPTS**

- Trigonometric equations often have multiple solutions. Ensure that you find all solutions that lie in the domain of interest.
- Quadratic trigonometric equations can often be solved by factoring.
- You will need to use the exact values for the trigonometric ratios of special angles and their multiples. Refer back to your unit circle if you forget.
- When solving for all solutions within the domain of interest, you will also need to make use of the CAST rule.

Examples:

1. Determine the exact solutions for the trigonometric equation $2 \cos x + 1 = 0$ in the interval $x \in [0, 2\pi]$.
2. Solve $(3 \cos x + 7)(-2 \sin x - 1) = 0$ on the interval $x \in [0, 2\pi]$.

3. Solve $2\sin(x)\cos(-x) = 2\sin(-x)\sin(x)$ on the interval $x \in [0, 2\pi]$.

4. Solve $\sin^2 x + \sin x = 6$ and find all solutions.